Single module identification – partial inputs

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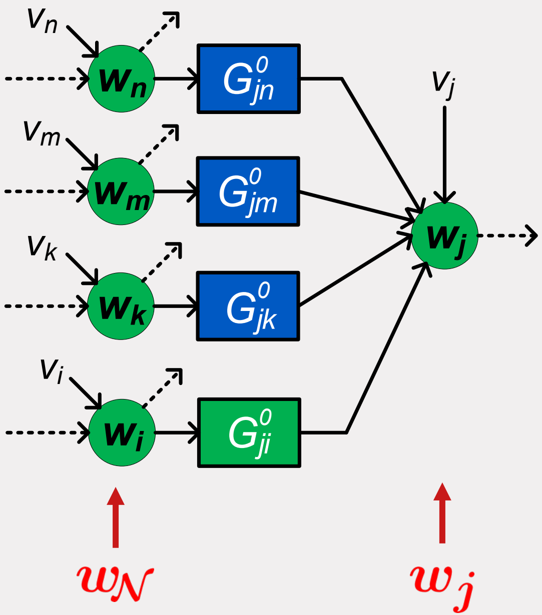
Single module identification

Full MISO situation:

Measure output w_j of target module and all in-neighbors $w_{\mathcal{N}}$ of w_j



Multi-input single-output identification problem

addressed by either a direct or indirect method

Question:

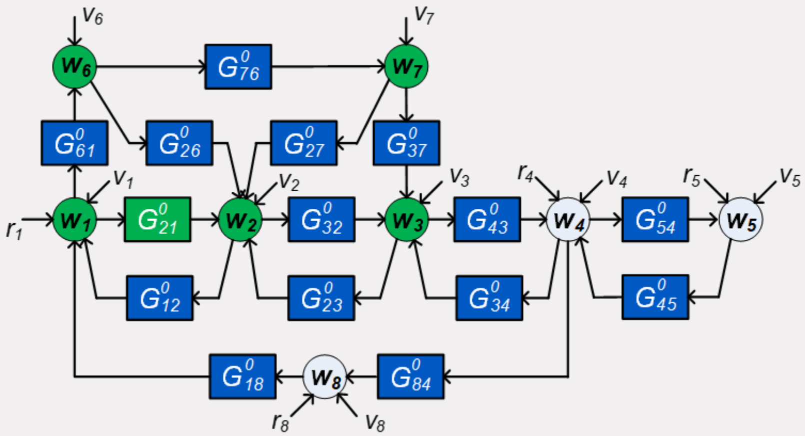
Can we reduce the number of input signals?

Do all in-neighbors of w_j need to be included?

Single module identification

4 input nodes to be measured:

Can we do with less?



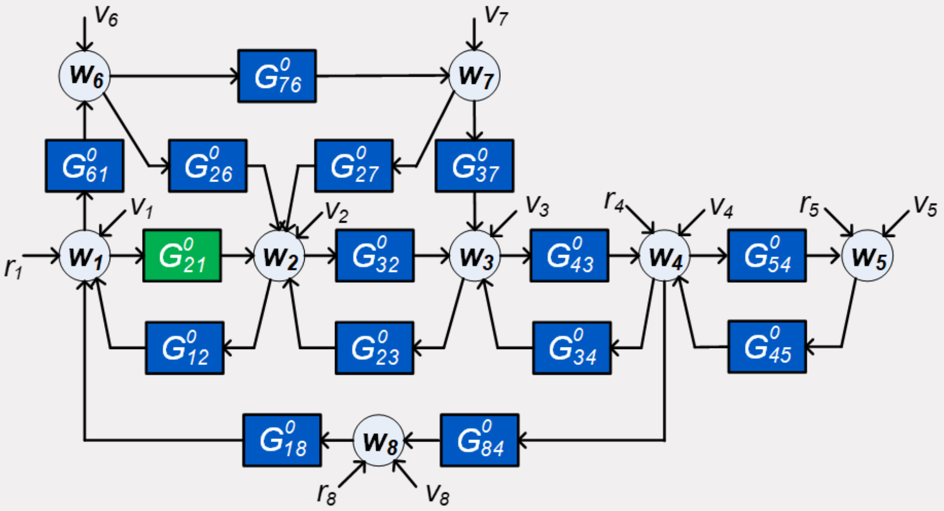
Network immersion [1]

- An **immersed network** is constructed by removing node signals, but leaving the remaining node signals invariant
- Modules and disturbance signals are adapted
- Abstraction through variable elimination (Kron reduction[2] in network theory).

[1] A. Dankers. PhD Thesis, 2014.

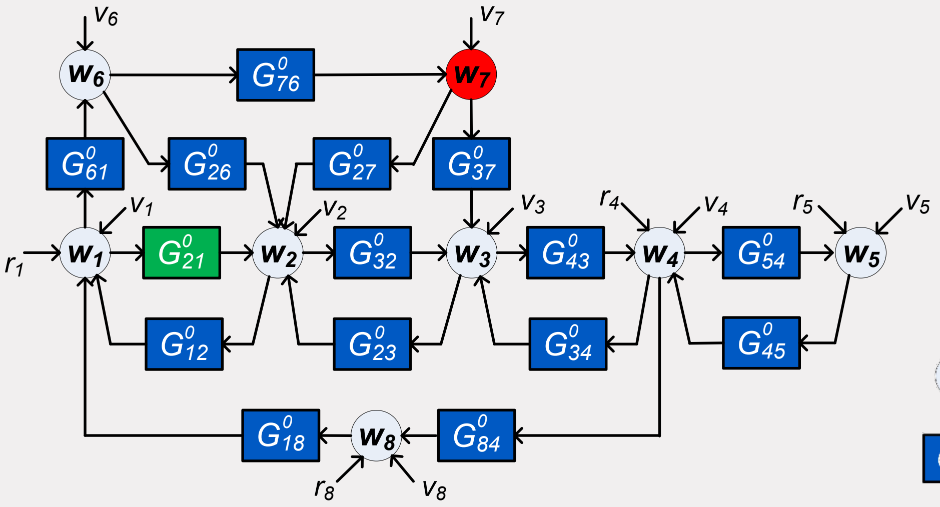
[2] F. Dörfler and F. Bullo, IEEE Trans. Circuits and Systems I (2013)

Immersion

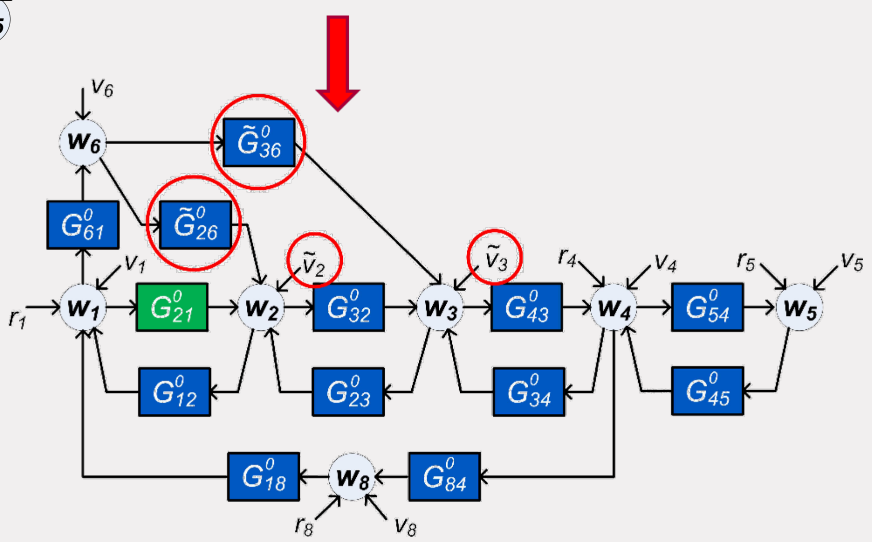


v_6
v_7
w_6
G^0_{76}
w_7
G^0_{61}
G^0_{26}
G^0_{27}
G^0_{37}
v_1
v_2
v_3
r_4
v_4
r_5
v_5
r_1
w_1
G^0_{21}
w_2
G^0_{32}
w_3
G^0_{43}
w_4
G^0_{54}
w_5
G^0_{12}
G^0_{23}
G^0_{34}
G^0_{45}
G^0_{18}
w_8
G^0_{84}
r_8
v_8

Immersion



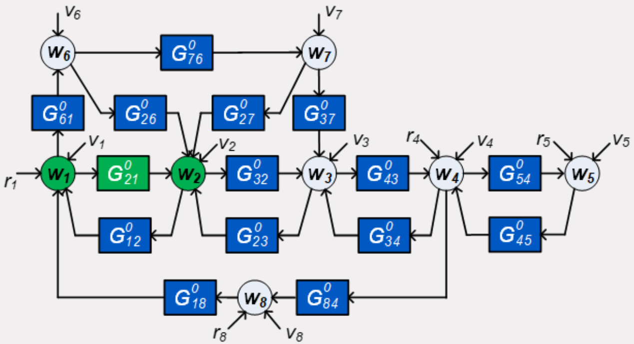
Immersing w_7

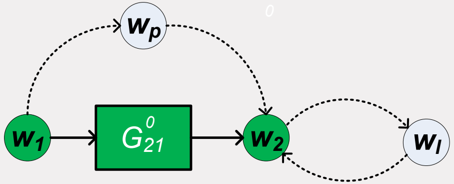


When does immersion leave G^0_{21} invariant?

Immersion

When does immersion leave G_{21}^0 invariant?





Proposition

Consider an immersed network where w_1 and w_2 are retained.

Then $\breve{G}_{21}^0 = G_{21}^0$ if

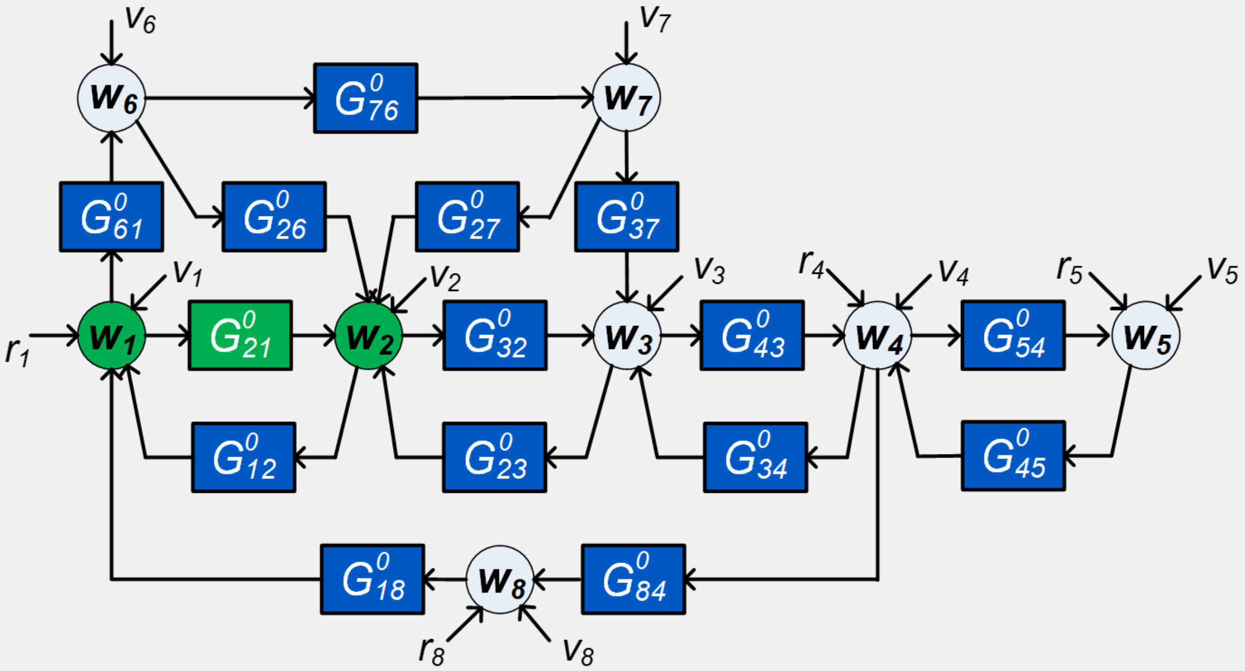
a) Every path $w_1 \rightarrow w_2$ other than the one through G_{21}^0 passes through a node that is retained. (parallel paths)

b) Every path $w_2 \rightarrow w_2$ passes through a node that is retained. (loops around the output)

A.G. Dankers et al., *IEEE Trans. Automatic Control, 61*, 937-952, 2016.

Single module identification

parallel paths, and loops around the output



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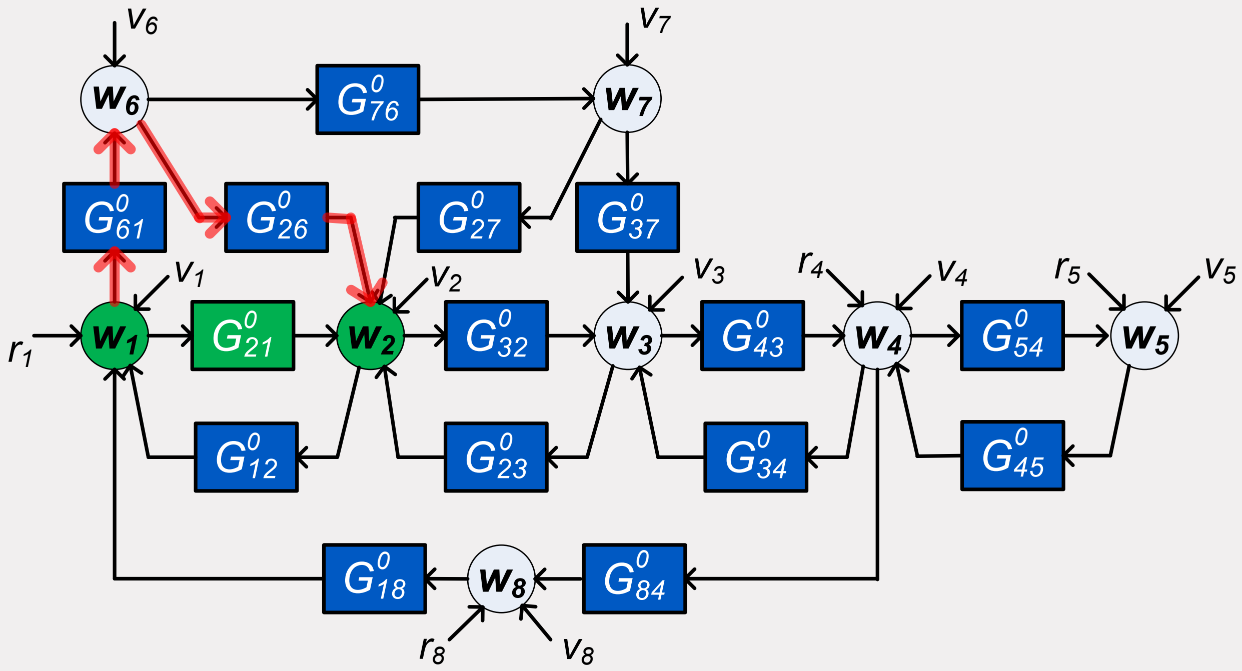
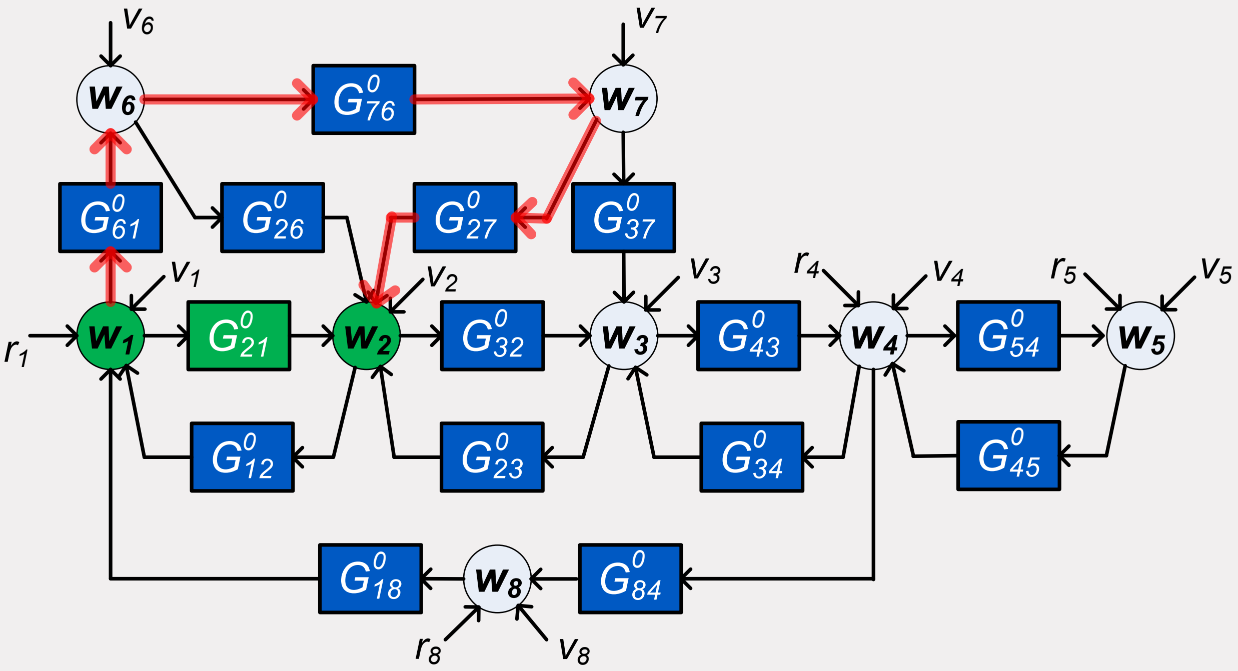
Single module identification

parallel paths, and loops around the output



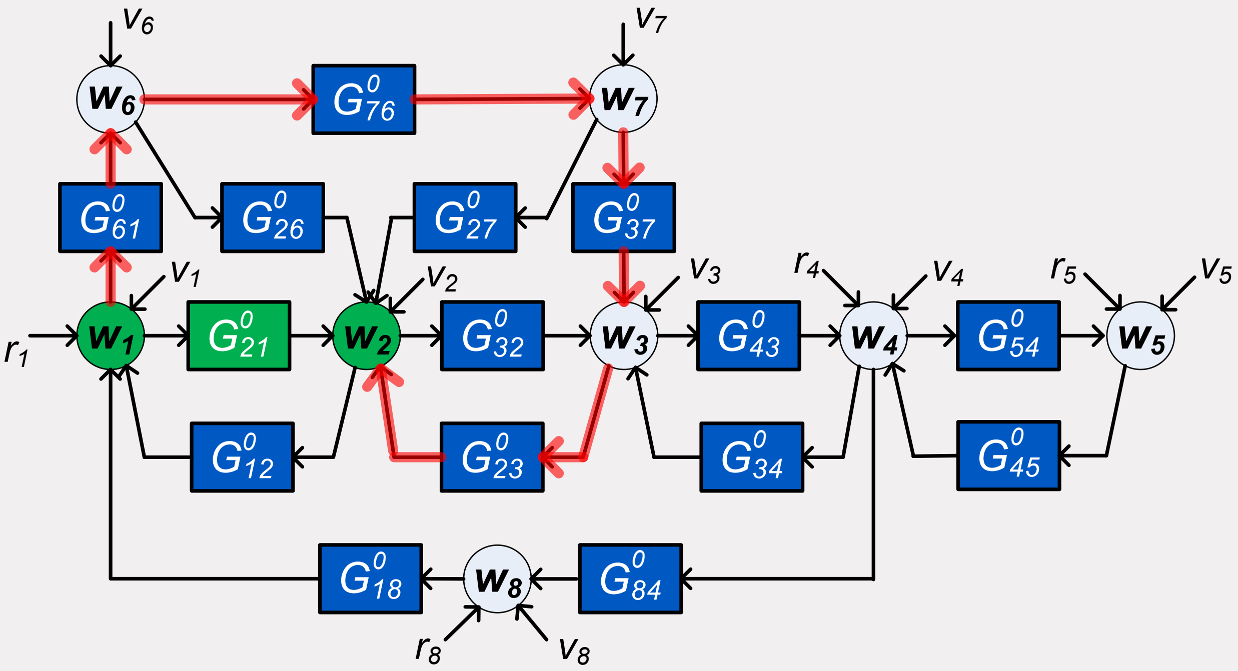
Single module identification

parallel paths, and loops around the output



Single module identification

parallel paths, and loops around the output



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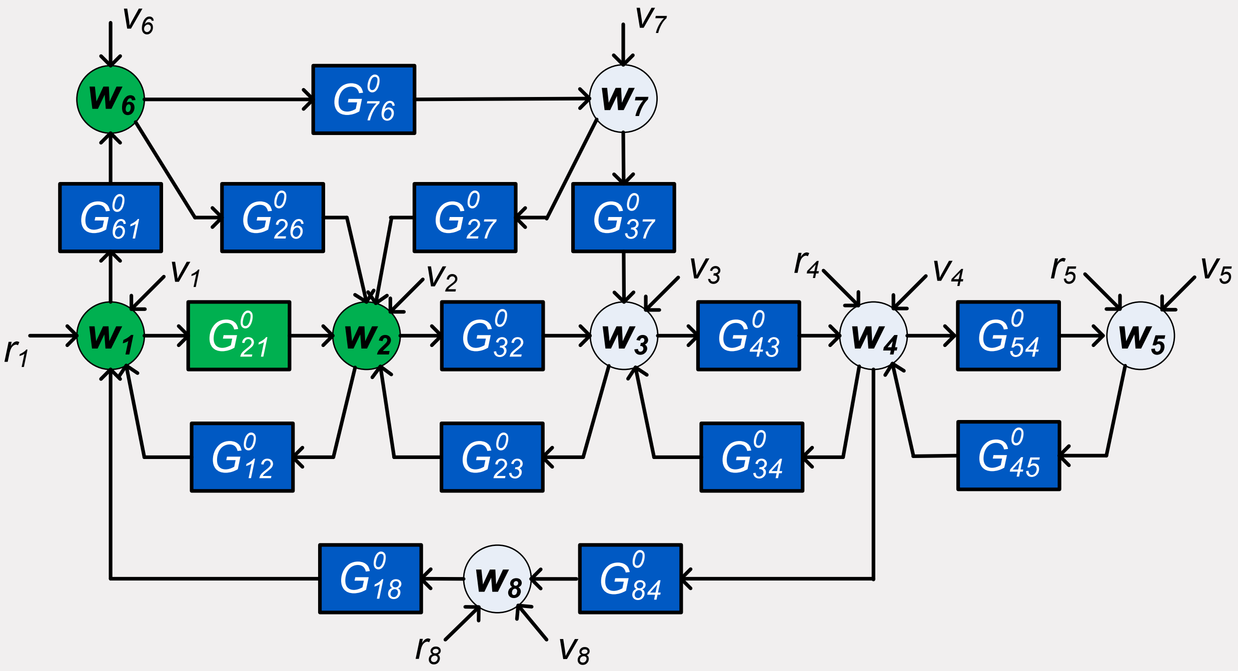
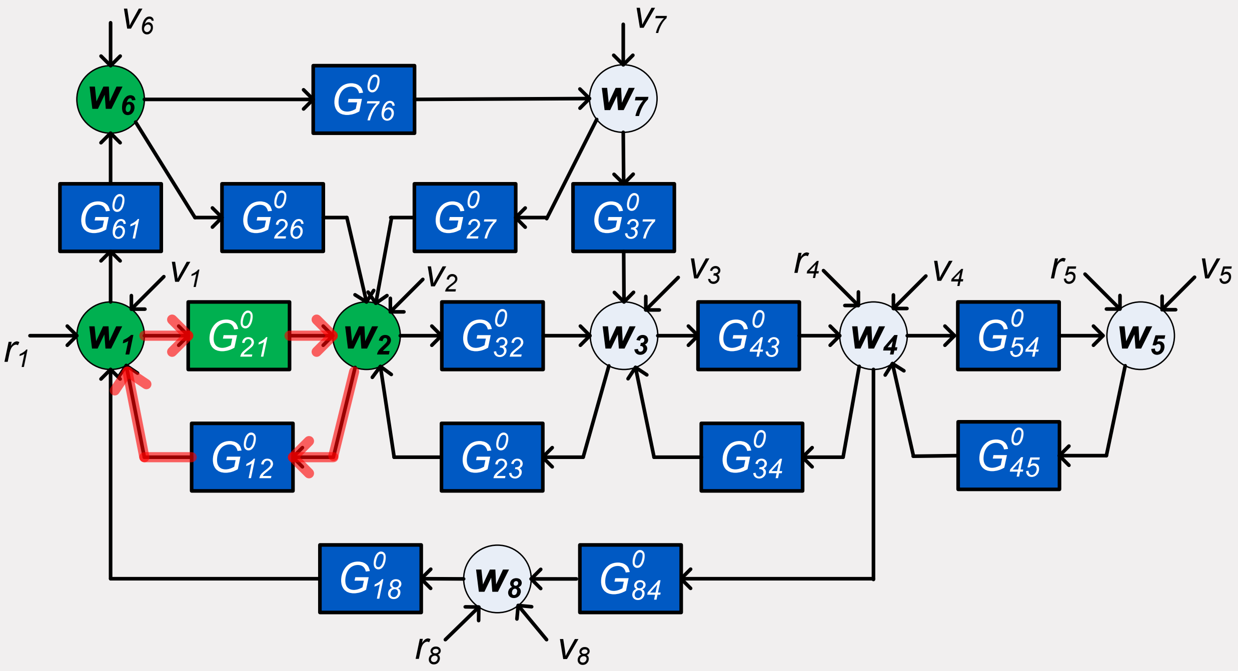
Single module identification

Choose w_6 as an additional input (to be retained)



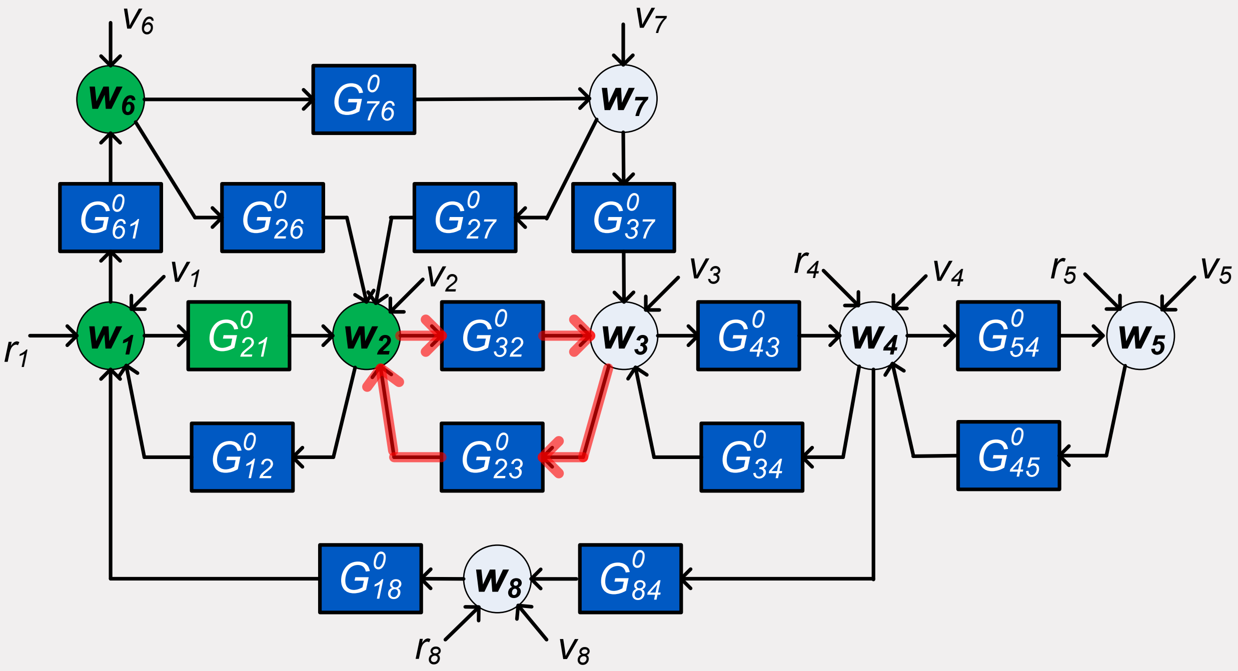
Single module identification

parallel paths, and **loops around the output**



Single module identification

parallel paths, and **loops around the output**



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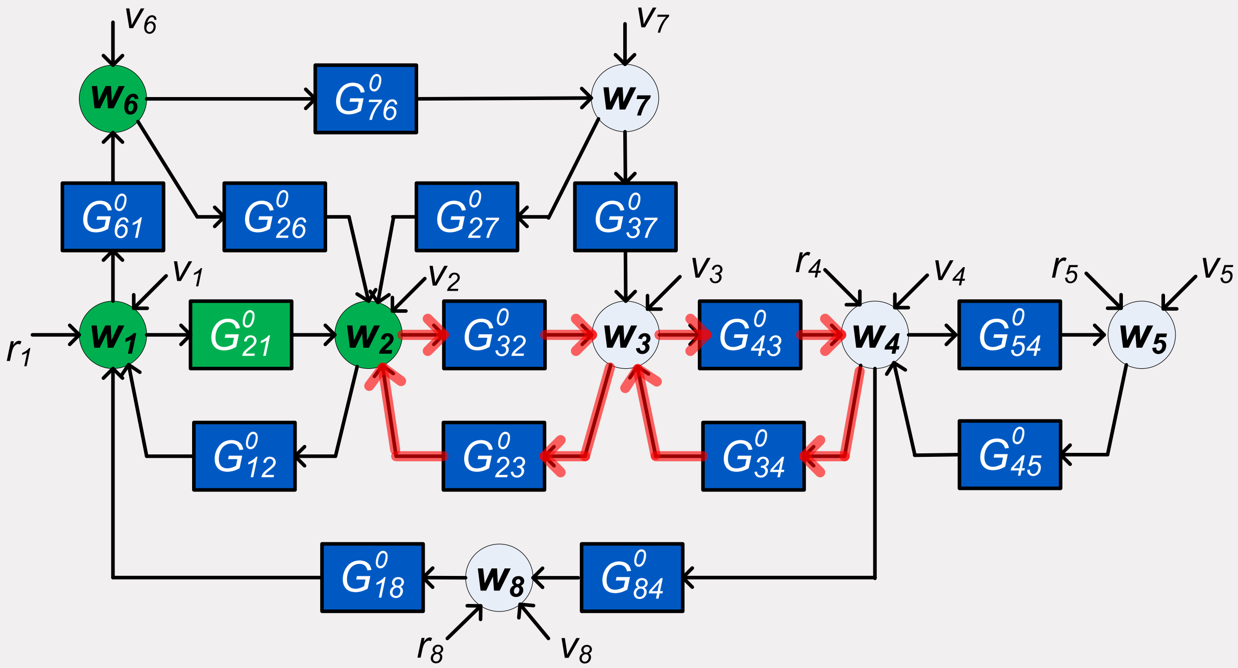
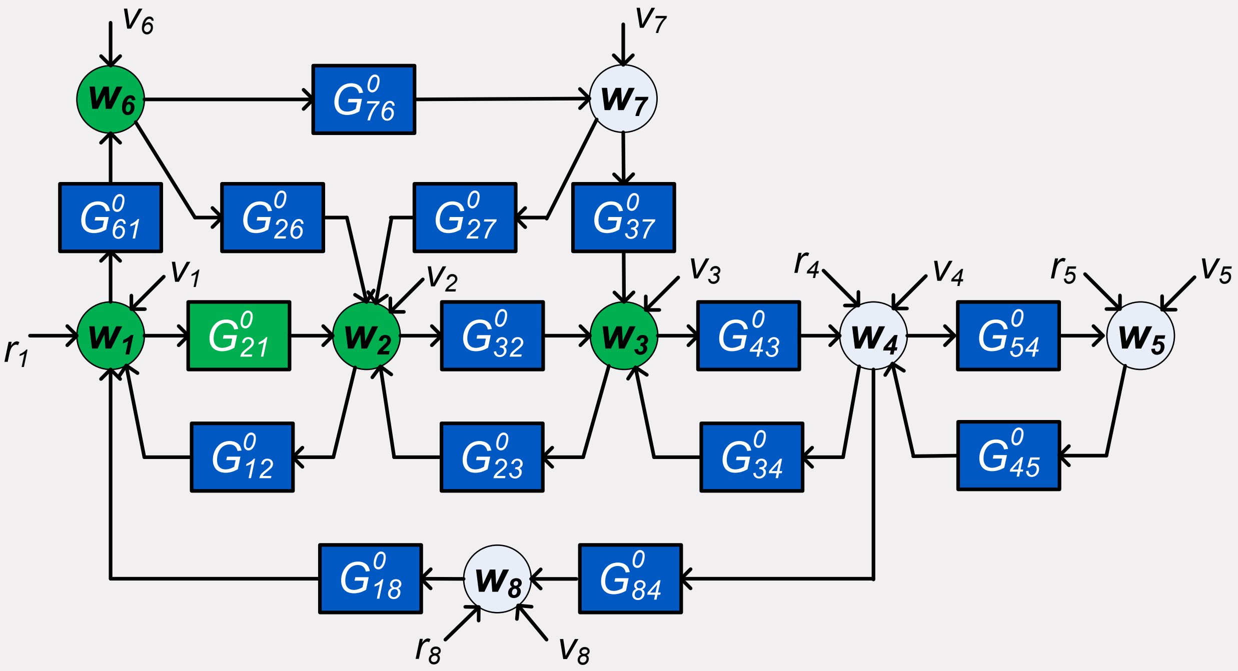
Single module identification

parallel paths, and **loops around the output**

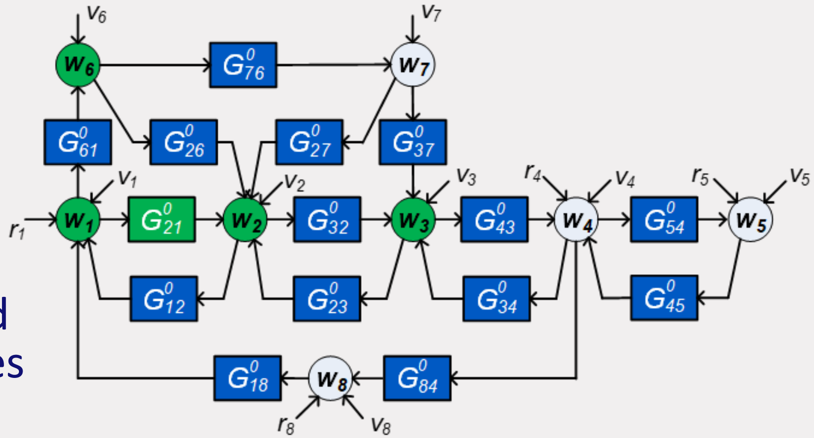


Single module identification

Choose w_3 as an additional input, to be retained



Single module identification



Conclusion:

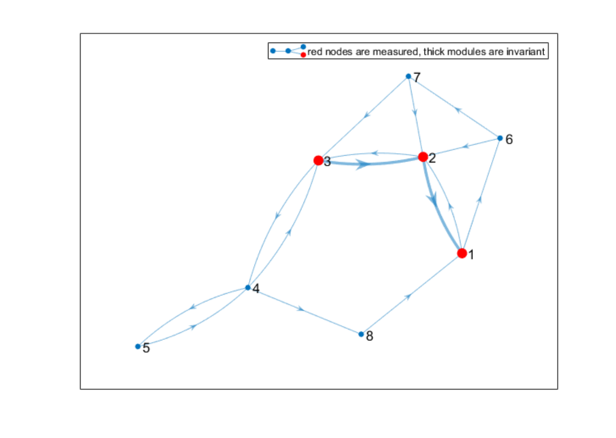
With a 3-input, 1 output predictor model, the module G_{21}^{0} remains invariant.

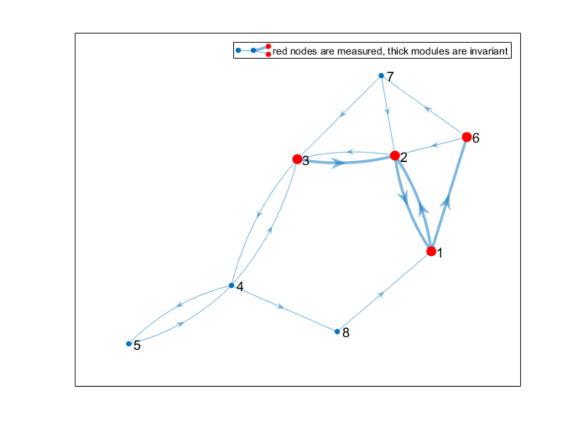
The **indirect method** can directly be applied to this reduced-input situation, and provides consistency

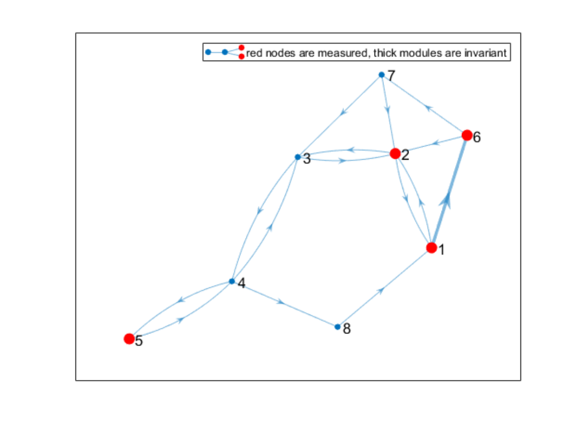
For the **direct method** the properties of the disturbances need to be further investigated

Invariant modules for a given set of measured nodes

For a selected set of measured nodes: algorithm determines
with modules remain invariant:

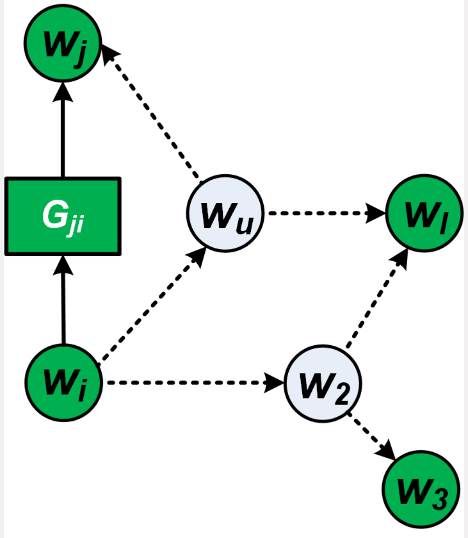






Generalization of immersion

Parallel paths and loops around the output can also be blocked by indirect measurements:



- In stead of measuring w_u we measure a descendant of w_u
- Every path from w_i to that descendant needs to be blocked too
- Indirect measurements may lead to non-proper modules

see e.g. Linder and Enqvist[1], Gevers et al.[2] , Weerts et al.[3]

[1] J. Linder and M. Enqvist. *Int. J. Control*, 2017.
[2] A. Bazanella, M. Gevers et al., CDC 2017.
[3] H. Weerts, J. Linder, M. Enqvist & PVdH, Automatica, 2020

Generalization of immersion - Abstraction

The applied reasoning is to exploit the degrees of freedom in the network representation:

$$\begin{bmatrix} w_{\tilde{\mathcal{S}}} \\ w_{\mathcal{L}} \\ w_{\mathcal{V}} \\ w_{\tilde{\mathcal{Z}}} \end{bmatrix} = \begin{bmatrix} G_{\tilde{\mathcal{S}}\tilde{\mathcal{S}}} & G_{\tilde{\mathcal{S}}\mathcal{L}} & G_{\tilde{\mathcal{S}}\mathcal{V}} & G_{\tilde{\mathcal{S}}\tilde{\mathcal{Z}}} \\ G_{\mathcal{L}\tilde{\mathcal{S}}} & G_{\mathcal{L}\mathcal{L}} & G_{\mathcal{L}\mathcal{V}} & G_{\mathcal{L}\tilde{\mathcal{Z}}} \\ G_{\mathcal{V}\tilde{\mathcal{S}}} & G_{\mathcal{V}\mathcal{L}} & G_{\mathcal{V}\mathcal{V}} & G_{\mathcal{V}\tilde{\mathcal{Z}}} \\ G_{\tilde{\mathcal{Z}}\tilde{\mathcal{S}}} & G_{\tilde{\mathcal{Z}}\mathcal{L}} & G_{\tilde{\mathcal{Z}}\mathcal{V}} & G_{\tilde{\mathcal{Z}}\tilde{\mathcal{Z}}} \end{bmatrix} \begin{bmatrix} w_{\tilde{\mathcal{S}}} \\ w_{\mathcal{L}} \\ w_{\mathcal{V}} \\ w_{\tilde{\mathcal{Z}}} \end{bmatrix} + \begin{bmatrix} u_{\tilde{\mathcal{S}}} \\ u_{\mathcal{L}} \\ u_{\mathcal{V}} \\ u_{\tilde{\mathcal{Z}}} \end{bmatrix} + \begin{bmatrix} v_{\tilde{\mathcal{S}}} \\ v_{\mathcal{L}} \\ v_{\mathcal{V}} \\ v_{\tilde{\mathcal{Z}}} \end{bmatrix},$$

- $i, j \in \tilde{\mathcal{S}}$ as well as other measured nodes
- nodes in $\mathcal{V}$ are not measured but indirectly observed by nodes in $\mathcal{L}$
- Remove $w_{\tilde{\mathcal{Z}}}$ by solving for the 4th equation.
- Remove $w_{\mathcal{V}}$ by solving for the 2nd equation.

$$\begin{bmatrix} w_{\tilde{\mathcal{S}}} \\ w_{\mathcal{L}} \end{bmatrix} = \begin{bmatrix} \breve{G}_{\tilde{\mathcal{S}}\tilde{\mathcal{S}}} & \breve{G}_{\tilde{\mathcal{S}}\mathcal{L}} \\ \breve{G}_{\mathcal{L}\tilde{\mathcal{S}}} & \breve{G}_{\mathcal{L}\mathcal{L}} \end{bmatrix} \begin{bmatrix} w_{\tilde{\mathcal{S}}} \\ w_{\mathcal{L}} \end{bmatrix} + \begin{bmatrix} \breve{u}_{\tilde{\mathcal{S}}} \\ \breve{u}_{\mathcal{L}} \end{bmatrix} + \begin{bmatrix} \breve{v}_{\tilde{\mathcal{S}}} \\ \breve{v}_{\mathcal{L}} \end{bmatrix}.$$

Determine conditions under which $\breve{G}_{\tilde{\mathcal{S}}\tilde{\mathcal{S}}} = G^0_{\tilde{\mathcal{S}}\tilde{\mathcal{S}}}$.

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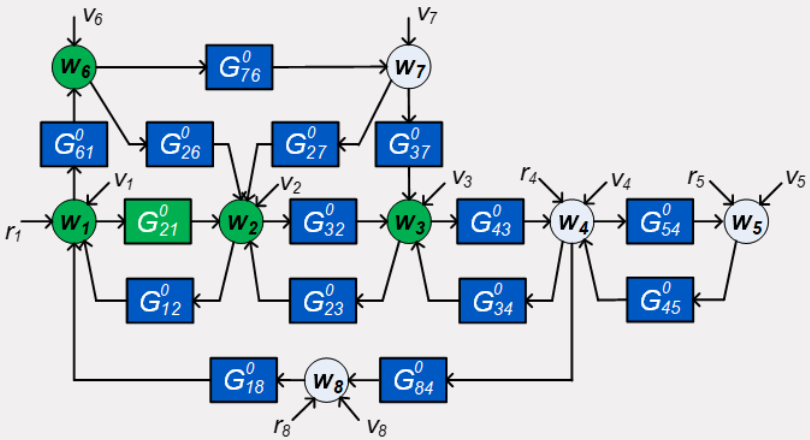
Single module identification

Conclusion:

With a 3-input, 1 output model we can consistently identify G_{21}^0

with an indirect method



For the direct method the condition that $\Phi_v(\omega)$ is diagonal needs to be sharpened in the situation that not all in-neighbors are taken as predictor inputs.

This is going to be addressed in the local direct method